An Alternative Explicit Six-Port Matrix Calibration Formalism Using Five Standards

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Abstract —**A new explicit six-port calibration method using five standards based on matrix formalism is developed. The new method has a number of advantages: There is no limitation on the measurements and in the six-port design to include a specific reference port for setting the power level during calibration and testing. The computation method is an explicit one and there is no need to use iterative procedures.**

I. Introduction

During the past decade, microwave measurement instrumentation has seen the emergence of the six-port as an alternative in some applications to conventional network analyzers [1], [2]. Using this technique, the magnitude and the phase of an unknown reflection coefficient are determined entirely by four relative power readings and the calibration constants at the measurement frequency. Different methodologies were used to relate the four power readings to the complex reflection coefficient, including linear formulations [3]–[9]. However the number of standard terminations as well as the computational effort to calibrate a six-port reflectometer needs to be further reduced. At first, seven standards and a linear procedure were used: in another case [5], four offset short circuits and a matched load were proposed [7]. Explicit six-port calibration methods using five standards were also suggested in [3], [4], and [8]. Later, four standards using an iterative method to calculate the reflection coefficient was used; this last method is limited to reflectometers which include a reference port [7].

This paper presents an alternative method to calibrate a six-port reflectometer using five standards and an explicit noniterative procedure based on matrix formalism. In order to verify the method, an experimental six-port reflectometer was calibrated over 2–4 GHz in 10 MHz steps using a junction previously reported in the literature [10]. Some of the advantages related to this new calibration are: a) matrix formalism is rapidly programmed and executed by computers, b) the computation method is an explicit one, and c) there is no limitation to dedicate one specific port to incident power level measurements: This last feature gives more latitude in the six-port junction design.

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II. Matrix Representation of Six-Port Reflectometer

Consider an arbitrary linear and time-stationary six-port network as shown in Fig. 1, with incident and reflected normalized a_i and b_i $(i=1,\cdots,6)$ waves, respectively. It has been shown by Engen [11] that the incident normalized waves at the four power detectors connected to ports 3, 4, 5, and 6 of a six-port can be represented by

$$b_e = M_e a_2 + N_e b_2. \tag{1}$$

M_e and N_e are complex parameters depending on the scattering parameters of the six-port junction and the reflection coefficients of the four detectors.

The reflection coefficient of the device under test connected to port 2 is given by:

$$\Gamma = \frac{a_2}{b_2}. \tag{2}$$

Substituting (1) into (2), we get p_e as a function of Γ:

$$p_e = A|b_2|^2\left[|N_e|^2 + |M_e|^2|\Gamma|^2 + M_e N_e^*\Gamma + N_e M_e^*\Gamma^*\right] \quad \text{for } e=3,\cdots,6 \tag{3}$$

with A a scalar parameter. Equation (3) can be represented as a scalar product of vectors[1] as follows:

$$p_e = A|b_2|^2 \boldsymbol{L}_e^T \boldsymbol{R} \qquad \text{for } e=3,\cdots,6 \tag{4}$$

where

$$\boldsymbol{L}_e = \begin{vmatrix} |N_e|^2 \\ |M_e|^2 \\ N_e^* M_e \\ N_e M_e^* \end{vmatrix} \quad \text{and} \quad \boldsymbol{R} = \begin{vmatrix} 1 \\ |\Gamma|^2 \\ \Gamma \\ \Gamma^* \end{vmatrix}.$$

By introducing the linear operator T, we can relate the power reading vector $\boldsymbol{P}$ to the reflection coefficient vector

[1] The following notation is used for any vector V and matrix A:
$\boldsymbol{V}_T$: a column vector;
$\boldsymbol{V}_T$: a row vector;
A: the transpose of matrix A;
A^{-1} the inverse of square matrix A.

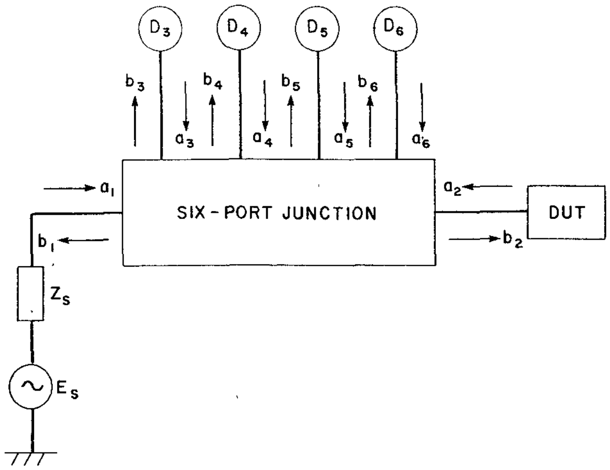

Fig. 1. Six-port reflectometer.

Γ by the following expression:

$$P = A|b_2|^2 LR = A|b_2|^2 LT^{-1}\Gamma = A|b_2|^2 C\Gamma = \alpha C\Gamma \quad (5)$$

where

$$\Gamma = \begin{vmatrix} 1 \\ |\Gamma|^2 \\ R(\Gamma) \\ I(\Gamma) \end{vmatrix} \qquad P = \begin{vmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{vmatrix} \qquad L = \begin{vmatrix} L_1^T \\ L_2^T \\ L_3^T \\ L_4^T \end{vmatrix}$$

$$T = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & -j/2 & +j/2 \end{vmatrix}.$$

Here L is a 4×4 complex matrix; C is a 4×4 real matrix; $R(\Gamma)$ is the real part of Γ; $I(\Gamma)$ is the imaginary part of Γ; and $p_j = p(e-2)$ $(j=1,\cdots,4)$.

The C matrix is a calibration matrix, and it represents the invariant properties of the measurement instrument including the six-port reflectometer. The value of $A|b_2|^2 = \alpha$ is the input power level of the device under test (DUT).

It was shown in [12] that the circle centers characterizing the design of an arbitrary six-port junction, with equivalent so-called q_i points in Engen notation, are related to the entries of the C matrix as follows:

$$q_i = -\frac{1}{2c_{i2}}(c_{i3} + jc_{i4})$$

and

$$|q_i|^2 = |c_{i1}/c_{i2}| \qquad \text{for } i=1,\cdots,4.$$

Referring to (5), the reflection coefficient vector can also be expressed as follows:

$$\Gamma = \frac{1}{A|b_2|^2} C^{-1} P = \frac{1}{A|b_2|^2} XP \quad (6)$$

where $C^{-1} = X$ is the inverse matrix of C.

The complex reflection coefficient is deduced by normalizing all the elements of the reflection coefficient Γ by the first element of the same vector. In this case the second, third, and fourth element of the Γ vector become

$$|\Gamma|^2 = (X_2^T P)/(X_1^T P) \quad (7a)$$

$$R(\Gamma) = (X_3^T P)/(X_1^T P) \quad (7b)$$

and

$$I(\Gamma) = (X_4^T P)/(X_1^T P) \quad (7c)$$

where X_i is a column vector with entries being the elements of the ith row of the X matrix, and $(X_1^T P)$ is equal to the power level exciting the device under test.

We can see from (7) that since the first element of Γ is unity the level in the computation is automatically set. This formalism gives a unified approach for different junction designs (with or without a specific reference port).

III. Calibration Technique Using Five Standards

To calibrate a six-port in a given frequency band it is necessary to determine the matrix X at a number of predetermined frequency points within the chosen bandwidth. Several linear methods have been published to calibrate the six-port reflectometer [3]–[6]. An alternative method based on the above matrix formalism and using five standard terminations is developed below.

Equation (5) can be written for four terminations with four reflection coefficients $\Gamma_k (k=1,\cdots,4)$ as follows:

$$P_k = \alpha_k C\Gamma_k \qquad \text{for } k=1,\cdots,4$$

where $\alpha_k = A|b_2|^2$.

An explicit expression for the last equation gives

$$\alpha_k(c_{11} + c_{12}|\Gamma_k|^2 + c_{13}R_k + c_{14}I_k) = p_{1k} \quad (8a)$$

$$\alpha_k(c_{21} + c_{22}|\Gamma_k|^2 + c_{23}R_k + c_{24}I_k) = p_{2k} \quad (8b)$$

$$\alpha_k(c_{31} + c_{32}|\Gamma_k|^2 + c_{33}R_k + c_{34}I_k) = p_{3k} \quad (8c)$$

$$\alpha_k(c_{41} + c_{42}|\Gamma_k|^2 + c_{43}R_k + c_{44}I_k) = p_{4k}. \quad (8d)$$

The normalization of p_{2k}, p_{3k}, and p_{4k} on p_{1k} gives

$$c_{21} + c_{22}|\Gamma_k|^2 + c_{23}R_k + c_{24}I_k = (p_{2k}/p_{1k})(c_{11} + c_{12}|\Gamma_k|^2 + c_{13}R_k + c_{14}I_{1k}) \quad (9a)$$

$$c_{31} + c_{32}|\Gamma_k|^2 + c_{33}R_k + c_{34}I_k = (p_{3k}/p_{1k})(c_{11} + c_{12}|\Gamma_k|^2 + c_{13}R_k + c_{14}I_{1k}) \quad (9b)$$

$$c_{41} + c_{42}|\Gamma_k|^2 + c_{43}R_k + c_{44}I_k = (p_{4k}/p_{1k})(c_{11} + c_{12}|\Gamma_k|^2 + c_{13}R_k + c_{14}I_{1k}) \quad \text{for } k=1,\cdots,4. \quad (9c)$$

Representing the vector reflection coefficients as a (4×4)

G matrix, we have

$$g_{k1} = 1$$
$$g_{k2} = |\Gamma_k|^2 \qquad \text{for } k = 1, \cdots, 4$$
$$g_{k3} = R_k$$
$$g_{k4} = I_k.$$

In addition, the four power readings related to the four standards used above, can be given in four (4×4) matrices as follows:

$$P_j = \text{Diag}(p_{j1}, p_{j2}, p_{j3}, p_{j4}), \qquad j = 1, \cdots, 4.$$

Representing the ith row of matrix C as a column vector $\boldsymbol{C}_i$, and using the above matrix notation, the four equations (9a) $(k = 1, \cdots, 4)$ can be written as

$$G\boldsymbol{C}_2 = P_2 P_1^{-1} G\boldsymbol{C}_1.$$

Therefore

$$\boldsymbol{C}_2 = G^{-1} P_2 P_1^{-1} G\boldsymbol{C}_1. \tag{10a}$$

The same treatment can be done for (9b) and (9c) to give

$$\boldsymbol{C}_3 = G^{-1} P_3 P_1^{-1} G\boldsymbol{C}_1 \tag{10b}$$

$$\boldsymbol{C}_4 = G^{-1} P_4 P_1^{-1} G\boldsymbol{C}_1. \tag{10c}$$

The choice of the first four standards must ensure that the G matrix is nonsingular and that p_{ik} can never vanish, unless $c_{i1} = c_{i2} = c_{i3} = c_{i4} = 0$, which is contrary to the definition of matrix C and the behavior of the six-port junction. Consequently the inverse of matrix P_k exists.

The determination of the three vectors $\boldsymbol{C}_2$, $\boldsymbol{C}_3$, and $\boldsymbol{C}_4$ requires that $\boldsymbol{C}_1$ be first known. From (10) $\boldsymbol{C}_1$ can be deduced as follows:

$$\boldsymbol{C}_1 = G^{-1} P_1 P_2^{-1} G\boldsymbol{C}_2 \tag{11a}$$

$$\boldsymbol{C}_1 = G^{-1} P_1 P_3^{-1} G\boldsymbol{C}_3 \tag{11b}$$

$$\boldsymbol{C}_1 = G^{-1} P_1 P_4^{-1} G\boldsymbol{C}_4. \tag{11c}$$

From (11) and using some matrix manipulations, it is easy to show that

$$\boldsymbol{C}_i = G^{-1} P_i P_j^{-1} G\boldsymbol{C}_j \tag{12}$$

for $i = 1, \cdots, 4$ and $j = 1, \cdots, 4$, with $i \neq j$.

Writing the three expressions of $\boldsymbol{C}_i$ for two different values of i, we can deduce that

$$P_1^{-1} G\boldsymbol{C}_1 = P_2^{-1} G\boldsymbol{C}_2 = P_3^{-1} G\boldsymbol{C}_3 = P_4^{-1} G\boldsymbol{C}_4 = \boldsymbol{V}_0 \tag{13}$$

where $\boldsymbol{V}_0$ is a (4×1) column vector.

Using (6) and substituting the expression for p_{ik} in (13), it can be seen directly that $\boldsymbol{V}_0$ is

$$\boldsymbol{V}_0^T = (1/\alpha_1, 1/\alpha_2, 1/\alpha_3, 1/\alpha_4)$$

where $\alpha_k (k = 1, \cdots, 4)$ is the input power level exciting the standard termination k when it is connected to the measuring plane of the six-port reflectometer.

The replacement of $P_j^{-1} G\boldsymbol{C}_j$ by $\boldsymbol{V}_0$ in (12) gives

$$\boldsymbol{C}_i = G^{-1} P_i \boldsymbol{V}_0 \qquad \text{for } i = 1, \cdots, 4. \tag{14}$$

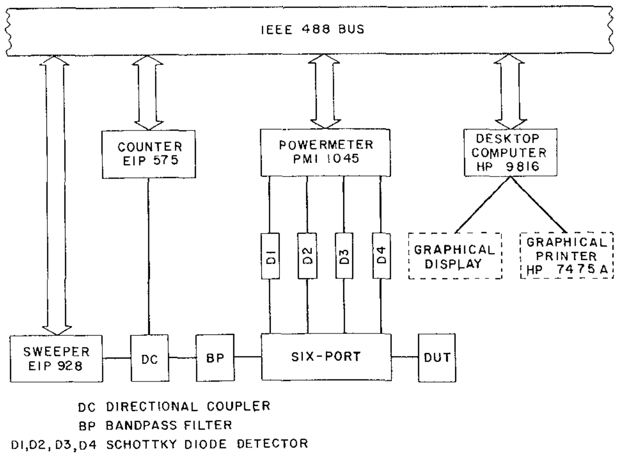

Fig. 2. Experimental block diagram of six-port reflectometer.

Equations (10) and (14) are completely equivalent and there is no need in the above treatment to assume that the six-port has a specific reference port.

In the Appendix it is shown that the vector $\boldsymbol{V}_0$ elements are obtained as the solution of the following linear system:

$$\boldsymbol{P}_0 = P\, \text{Diag}\left[(G^{-1})^T \Gamma_0 \right] \boldsymbol{V}_0 \tag{15}$$

where $\boldsymbol{P}_0$ is the power readings vector of the nonideal matched load (fifth standard), Γ_0 is the reflection coefficient vector of the fifth standard, and $P = (\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \boldsymbol{P}_4)$ is a 4×4 power matrix.

From (14) and (15), it is seen that the determination of the calibration matrix needs an explicit matrix product without any iterative computation procedures.

IV. Experimental Results

For an ideal six-port reflectometer it was shown in [12] that the entries to the C matrix c_{ij} are related by the following equation:

$$(c_{i4})^2 + (c_{i3})^2 - 4c_{i1}c_{i2} = 0 \qquad \text{for } i = 1, \cdots, 4. \tag{16}$$

For a practical six-port reflectometer, an error in the actual calibration parameters c_{ij} produces a nonzero value for the above equation. An evaluation of inaccuracy on the calibration matrix can be suggested as follows:

$$E = \sum_{i=i_0}^{4} |E_i| = \sum_{i=i_0}^{4} \left| (c_{i3})^2 + (c_{i4})^2 - 4c_{i1}c_{i2} \right| \tag{17}$$

with $i_0 = 1$ for a six-port without a reference port, $i_0 = 2$ for a six-port with a reference port, and $|E_i|$ the estimated error related to the actual calibration parameters.

The calibration algorithm has been implemented on a desk-top computer (HP 9816) and applied to a six-port junction [10]. Three positions of a precision Narda 901NF type sliding short (separated by a distance of $\lambda g/8$ of a given calibration frequency value), a Wiltron matched load (type 28A50-1), and a shorted 6 dB attenuator as a fifth standard were used to calibrate the reflectometer over a 2–4 GHz range in 10 MHz steps. The block diagram of the six-port reflectometer is shown in Fig. 2. Tables I and II show that the values of the error function E_i are very

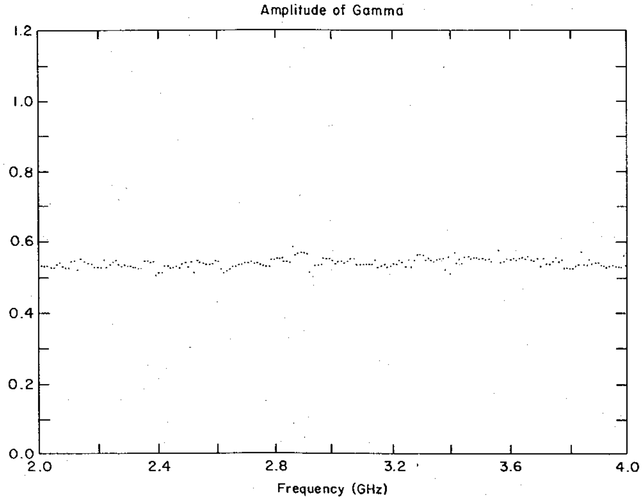

Fig. 3. Direct return loss measurement of a 3 dB attenuator over 2–4 GHz in 10 MHz steps.

TABLE I
EXPERIMENTAL RESULTS OF SIX-PORT REFLECTOMETER

(a)						(b)		
EXPERIMENTAL CALIBRATION MATRIX						MEASURED VALUE / EXPECTED VALUE		
j / i	1	2	3	4	E_i	standard	amplitude	phase (degree)
1	.1096	-.0013	-.0060	-.0171	.0009	open circuit	1.000/1.000	reference plane
2	.0679	.0106	-.0663	.0160	.0018	short circuit	1.019/1.000	-180.913 / -180.000
3	.0373	.0174	.0295	.0182	-.0014	----	----	----
4	.0323	.0230	.0114	-.0573	.0004	attenuator* in db	-19.573 / -19.086	----

(a) Calibration matrix of 4 GHz.
(b) Comparison of test results with results obtained on HP8510T and nominal values for short and open circuit.
*The expected value is the value measured with an HP8510T at Northern Telecom, Canada (March 1986).

TABLE II
EXPERIMENTAL RESULTS OF SIX-PORT REFLECTOMETER

(a)						(b)		
EXPERIMENTAL CALIBRATION MATRIX						MEASURED VALUE / EXPECTED VALUE		
j / i	1	2	3	4	E_i	standard	amplitude	phase (degree)
1	.3381	.0023	-.0211	.0174	-.0023	----	----	----
2	.2326	.0999	-.0240	-.1299	-.0755	short circuit	1.013 / 1.000	reference plane
3	.1090	.0512	-.0998	.0982	-.0027	coax air line+ S.C. (10.35cm)	1.003 / 1.000	+179.868 / +180.000
4	.0898	.0291	.1161	.0069	.0031	----	----	----

(a) Calibration matrix at 2.89 GHz.
(b) Test results of a coaxial air line (10.35 cm).

small. This is an indication that the actual calibration matrix describes the response of the six-port reflectometer very well. Table I shows the measured magnitude and phase of an open circuit, a short circuit, and the return loss of a 10 dB attenuator at 4 GHz. The measured phase value of the open circuit is taken as the reference plane. Table II shows the measured magnitude and phase of a short circuit and a coaxial air line (10.35 cm) terminated by a short circuit at 2.89 GHz. At this frequency a coaxial air line of 10.35 cm length has a 4π phase angle. In this case the measured phase value of a short circuit is taken as the reference plane. The phase values found for open and short circuit conditions in Tables I and II are within 1 percent of expected values. Fig. 3 shows the return loss measurement of a 3 dB attenuator over 2–4 GHz in 10 MHz steps.

V. CONCLUSIONS

An alternative explicit six-port calibration method using five standards based on matrix formalism is presented and verified by experimental results. It is shown that the new method has the following advantages:

1) There is no limitation that the reflectometer must include a specific reference port to monitor the power level; this gives more latitude in designing the six-port junction.
2) The calibration method is completely based on matrix formalism; this allows rapid calculations and reduces the computational effort.
3) In addition the computation method is an explicit one (not requiring iteration).

APPENDIX

Consider a load with a reflection coefficient vector $\boldsymbol{\Gamma}_0$. The power vector $\boldsymbol{P}_0$ is related to $\boldsymbol{\Gamma}_0$ by

$$\boldsymbol{P}_0 = \alpha_0 C \boldsymbol{\Gamma}_0. \qquad \text{(A1)}$$

An explicit form of the last equation is

$$p_{i0} = \alpha_0 \boldsymbol{C}_i \boldsymbol{\Gamma}_0 \qquad \text{for } i = 1, \cdots, 4. \qquad \text{(A2)}$$

Substituting (14) in (A2), one can get

$$p_{i0} = \alpha_0 \left(G^{-1} P_i V_0\right)^T \boldsymbol{\Gamma}_0 = \alpha_0 V_0^T P_i \left(G^{-1}\right)^T \boldsymbol{\Gamma}_0 \qquad \text{(A3)}$$

where $V_0^T = (1/\alpha_1, 1/\alpha_2, 1/\alpha_3, 1/\alpha_4)$ and P_i is a diagonal matrix. Then $(P_i)^T = P_i$. If we assume that $G^{-1} = (\boldsymbol{G}_1, \boldsymbol{G}_2, \boldsymbol{G}_3, \boldsymbol{G}_4)$, where $\boldsymbol{G}_i$ is the ith column vector of matrix G^{-1}, it is easy to obtain that

$$p_{i0} = \alpha_0 V_0^T \left(p_{i1} \boldsymbol{G}_1^T \boldsymbol{\Gamma}_0;\ p_{i2} \boldsymbol{G}_2^T \boldsymbol{\Gamma}_0;\ p_{i3} \boldsymbol{G}_3^T \boldsymbol{\Gamma}_0;\ p_{i4} \boldsymbol{G}_4^T \boldsymbol{\Gamma}_0 \right)^T.$$

Therefore

$$p_{i0} = \sum_{j=1}^{4} \frac{\alpha_0}{\alpha_j} p_{ij} \boldsymbol{G}_j \boldsymbol{\Gamma}_0 \qquad \text{for } i = 1, \cdots, 4. \qquad \text{(A4)}$$

In more compact form, the last equation can be rewritten as

$$\boldsymbol{P}_0 = P \operatorname{Diag}\left(\left(G^{-1}\right)^T \boldsymbol{\Gamma}_0\right) \cdot \alpha_0 V_0$$

where $P = (\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \boldsymbol{P}_4)$ is a 4×4 power matrix. The last linear system can be solved for $(\alpha_0 V_0)$. The term α_0 can be made equal to unity because the relative value of α_k is important, not the absolute one.

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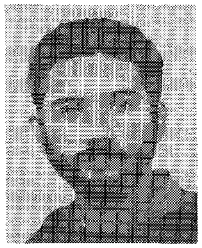

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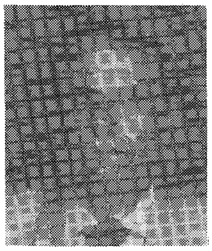

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